Coordinating in the haircut

A model of sovereign debt restructuring in secondary markets

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Abstract

Over the last two decades, countries that default on their debts increasingly have had to confront atomistic bondholders when engaging on restructuring negotiations. According to data, nature of creditors impacts on restructuring results, reducing investors concession to the govenrment in default. In this paper, we present a model to study the determination of haircut for defaulted debt when bondholders play a coordination game. We refine multiplicity with a global games approach and find that new market setting introduces an additional constraint to the government which then compresses the asked concession in order to increase the probability of program's acceptance. We run simulations with calibrated parameters and find that coordination costs account for a significant portion of the haircut reduction (10-30%) observed after sovereign debt disintermediation process.

Keywords: Sovereign debt; Restructuring; Secondary market; Coordination; Global games.

1 Introduction

By the end of the 1980s, emerging sovereign creditors' changed drastically: large international banks, were increasingly substituted with atomistic bondholders at global capital markets. The transition added a coordination friction amongst creditors to the processes of issuing, pricing, trading, defaulting and particularly renegotiating debt. Now the result in each process for every creditor depends on the decisions all others simultaneously make.

Three milestones paved the way for this transition (Andritzky, 2006). First, the establishment in the eighties of a high yield market to orderly trade risky instruments. Second, the Brady Plan in 1989 securitized most emerging economies' defaulted debt and contributed with extra volume to the referred high yield market. Third, increasing capital markets liberalization fueled flows of investments allocating in assets abroad. As a consequence, for example, private stocks of sovereign debt in emerging markets countries that split 80 to 20 per cent in 1980 between bank loans and bonds, inverted to 74 to 26 per cent bonds and banks loans in 2000 (figure 1a). In the same direction, defaulted and restructured instruments became increasingly bonds (figure 1b).

This change added a coordinating friction amongst creditors in all debt related processes. Yet, the sign of these effects was not clear at the beginning. Regarding restructuring, for instance, Peterson (1999) argues that the process might become "much harder" for the higher number of investors and its geographical dispersion. Reinhart and Rogoff (2009) similarly expect a more difficult process but with less frequency as governments would even try to avoid defaulting in such difficult conditions.

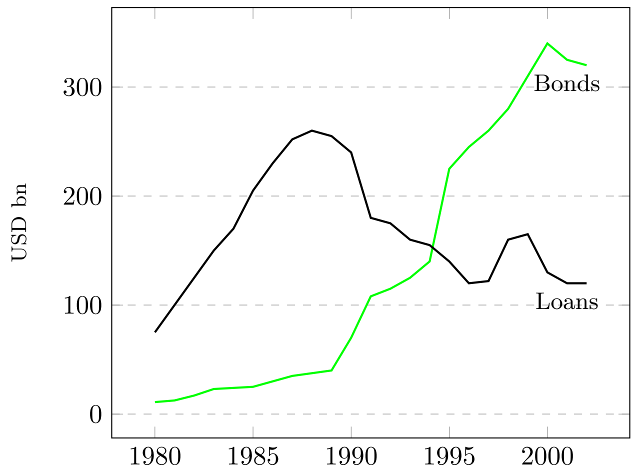
Quick data[1] inspection suggests that this new feature effectively did impact on restructuring process. Average haircut in 1998-2014 period reached 66% in the case of banks and 39% in the case of bonds with 19 and 21 cases respectively[2]. Indeed, after controlling for time effects (and economy size and GDP per capita), we find that the *haircut*[3] reduces consistently once bonds instead of loans are restructured. Broadly speaking, the effect averages -45% impact on haircut. (See appendix A).

In this paper we propose a model to determine haircut when defaulted debt is trading at secondary markets. The solution consists of an equilibrium in the strategic interaction between a constrained sovereign and a continuum of bondholders playing a coordination game of strategic complementaries. Modeling

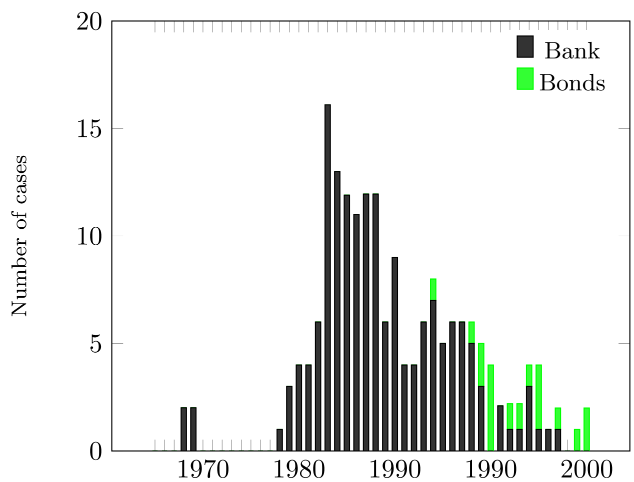
[1] Cruces and Trebesch restructuring database updated (2014).

[2] Bai and Zang (2012) document a 14% difference with banks' haircut above bonds' using 1990-2012 data in Benjamin and Wright (2008) haircut estimation.

[3] The restructuring plan can include one or many instruments such as increasing maturities, buybacks, cash tenders, bond exchanges amongst others; the total equivalent percentage loss on investment is summarized in "haircut".



Panel (a)



Panel (b)

Figure 1: (a) Structure of external public debt in emerging market countries (stocks of privately held debt). (Borensztein et al., 2004). (b) Finalized restructurings per year. (Anritzky, 2012).

bondholders decisions as in a simultaneous coordination game brings multiple equilibria to the results. We apply global games approach to refine multiplicity in terms of a fundamental threshold that identifies bondholders' decision zones. We find that the coordination feature actually restricts government's negotiating power reducing haircut even when the atomistic nature of creditors would suggest the opposite. Simulation exercise to compare model results with a coordination-free case (using Nash bargaining with all negotiation power in government) obtains consistently that coordination restricts haircut by 0-10 per cent depending on parameters values.

When defaulted debt is traded at secondary markets, the renegotiation confronts government in one side against reportedly many thousands of retail bondholders in the other.[4] With few experiences of successful representative groups (Das et al., 2012), current scale demands embedding coordination effects in the restructuring proposal in order to abbreviate the costly and possibly intractable negotiation process. This entails governments anticipation to each investor assessing both, the proposal and the others' assessment of it for an efficient solution. Understanding the effects of coordination in equilibrium may lead towards more efficient proposals both in terms of time spending and acceptance levels.

This paper contributes to the broad literature on sovereign debt, after the seminal paper Eaton and Gersovitz (1981) on the idiosyncrasy of sovereign borrowing and endogenous default. Following that line, Aguiar and Gopinath (2006) models default with business cycles and Arellano (2008) explains the relation between default and income fluctuations inside a small open economy. In both cases, restructuring occurs randomly and when it happens sovereign repays 100% of the debt. Yue (2010) endogenizes restructuring process using Nash bargaining to get the negotiated haircut. This feature requires to pre set not observable 'bargaining power' in order to replicate restructuring outcome, in her case, Argentina's debt crisis. In this paper we introduce global games approach to analyze the negotiating process in detail. Two interesting features are delivered by this methodology. First, the equilibrium haircut is completely endogenized. Second, it does not rely on non observable variables as bargaining power in Nash bargaining, and in contrast, the result derives from agents' interaction (government and bondholders) and their expectations on economic fundamentals.

Market disintermediation in sovereign debt has been signaled by some authors[5] and studied as the object in other articles. Between the latter, Bai and Zhang (2016) and Bi et al (2016) concentrate on the time reduction during the negotiating process from one environment to the other. In the first paper, secondary market avoided the endless processes by which the government tried to screen

[4] As were the cases of Dominica (2004), Pakistan (1999), Uruguay (2003), Seychelles (2009) and even hundred thousands such as the cases of Ukraine in 2000 (100 thousands) or Argentina in 2005 (600 thousands), (Andritzky, 2006).

[5] Andritzky (2006), Das et al (2012), De Brun and Della Mea (2003)

bank's outside option for defaulted debt. In the second paper contract innovations mainly, account for time savings in the process. We contribute to these authors by directly addressing other disintermediation effect: the observed reduction in restructuring outcome. We argue that this new market setting requires the coordination of atomized unconnected bondholders which entails a cost for the government that reflects in lower haircuts.

In addition, this paper relates to global games literature proposed in Carlsson and van Damme (1993) where information constraints are used to solve multiplicity in coordination games. They demonstrate that by introducing some noise in private information, players are forced to estimate others' participation in terms of fundamental distribution and that finally allows to solve equilibrium multiplicity by iterative deletion of strictly dominated strategies.

Finally, we would like to introduce a pair of comments about the model described below. First, we assume homogeneous agents which results in a solution with full or none agreement. This simplifying assumption, however, works properly to explain massive retail behavior, and allows us to work with *plain-vanilla* instruments (with no contract amendments). On the other hand, it prevents the possibility of interior equilibria and as a consequence the prediction of participation rate. Second, we are modeling decisions after default (which is the initial state), so we do not include sovereign's options' assessment previous to declaring it.

This paper is organized as follows. In section 2 we present the model. The structure advances through successive specifications: a complete information approach with multiple equilibria and a global games approach which introduces incomplete information to refine multiple equilibria. In section 3 we include the results which correspond to comparative statics, an econometric assessment of the model, and an estimation of coordination costs. Section 4 contains the main conclusions and further analysis roads.

2 Model

We model the haircut of defaulted debt traded at secondary market and the equilibrium participation of bondholders in the restructuring process as the equilibrium outcomes of a four stages game in which agents interact strategically (we define a stage as a period when the nature or an agent plays or makes a decision). There are two kinds of agents, the government (or sovereign) who defaulted on a set of outstanding debt and a continuous of agents holding one unit of defaulted bond each. Bondholders' decisions are modeled within a coordination game which entails multiple equilibria with complete information. We introduce incomplete information à la Morris and Shin using then global games approach as an equilibrium selection device. This setting allows us to

Government	Government	Nature	Investors	Government
Declares default	Determines and releases h	Draws and releases a realization of θ	Accept or reject	Accepts and pay or rejects and keeps on default
	Stage 1	Stage 2	Stage 3	Stage 4

Table 1: Time line

identify a threshold in economic fundamental that determines strict dominance regions ruling investors' decisions. Finally we compare our results with a non-coordination haircut estimated with Nash Bargaining to assess coordination costs and its response to model parameters.

2.1 The game in the time line

In table 1 we present the flow of the model to ease the reading. At stage zero, government defaults on a subset of total outstanding bonds.[6] In the first stage, government proposes a haircut h, corresponding to the concession asked to creditors of defaulted debt. In the following stage, nature draws a value for random variable θ which represents sovereign's current payment capacity, which is informed to all agents in the model. At the third stage, a continuum of creditors with a unit face value bond each, observe both the announcement h and fundamental θ, and determine individual binary action a_i of acceptance (rejection) of government proposal. In the last stage, government knowing θ, collects aggregate investors decisions (ℓ represents the proportion of accepting bondholders) and determines the result of the negotiation. The process ends at this stage in any case. If acceptance rate reaches a required threshold (in line with government's constraint), government pays bondholders and holdouts and exits default; otherwise, it exits negotiation without switching the state. We will solve this game using backwards induction from the last stage to the first one.

[6] These are plain-vanilla contracts, with no special provisions in the event of default.

2.2 Agents' payoffs

2.2.1 Sovereign

Government's information at stage four, includes economic fundamental θ and the aggregate acceptance rate $\ell \in [0,1]$ for the proposed program. We will denote this set with $\mathcal{I}^4_{gov} = \{\theta, \ell\}$, using super scripts to index the stage and under scripts to index the agent of reference. θ represents sovereign's current payment capacity. It is drawn by nature from a uniform density with boundaries $\check{\theta}, \hat{\theta} \in \mathbb{R}$ and would be applied as follows: the share ℓ of creditors whom accepted the proposal would receive $1-h$ for unit of bond and the share $1-\ell$ of rejecting creditors (or holdouts) would receive $\nu > 0$[7]. Haircut h represents the total equivalent percentage loss on investment for bondholders (as in Sturzenegger and Zettlemeyer (2008)).

Expression in (1) describes government's normalized payoffs after default G, as a function of acceptance rate, haircut and observed payment capacity. In the first line, government proposed a restructuring plan with haircut h, gathering acceptance level ℓ. As it exceeds some required threshold ℓ^* renegotiation process ends: sovereign pays bondholders as announced (h for those participating and ν for those rejecting the plan) and exits default receiving a boost of $\xi \in [0,1]$ in payment capacity θ.[8] Such impulse could derive from recovering access to capital markets, ease of international sanctions, bailout funds received, implementation of structural reforms, political and financial distress amongst others. In the second line, proposal gathers a low market acceptance (below governments required threshold), renegotiation fails and government remains in current state θ.

$$G(\ell, h, \theta) = \begin{cases} \theta(1+\xi) - [\ell(1-h) + (1-\ell)\nu] & \text{if } \ell \geq \ell^* \\ \theta & \text{if } \ell < \ell^* \end{cases} \tag{1}$$

Clearly, for the government to engage in a restructuring plan, it is needed that $\theta\xi \geq \ell(1-h-\nu) + \nu$ in (1), which implies that post restructuring assets gain has to exceed total compromised outcome.

We assume exogenous holdouts payment $\nu \in (1-h, 1]$[9]. When $\nu = 1$ government pays the full bond value to the share of holdout investors $1-\ell$. Where $\nu = 1-h$, government would pay the same amount to all debtors whether they accept or not, so that participation rate would turn irrelevant. In this case,

[7] Using data from US corporate debt, holdout premium against early settlers situated at 11% in 115 restructurings between 1992 and 2000 (Fridson and Gao, 2002) and at 30% in 202 restructurings between 1980 1992 (Altman and Eberhart, 1994).

[8] Annual median growth in restructuring countries increases from 1.5% previous the final agreement to 4 to 5% after it (Das, et al (2012) on Trebesch (2011) data set).

[9] This total value would eventually be settled by the government or a judge in court.

government could announce $h = 1$ and yet exit default without any repayment to bondholders. We then set $\nu > 1 - h$, to avoid this trivial scenario.

Note that when determining the haircut at stage one government's information set is $\mathcal{I}^1_{gov} = \{\emptyset\}$ forcing it then to appeal to the distribution of θ then.

2.2.2 Bondholders

A bondholder i from a continuum of measure one set with one unit bond chooses an individual action $a_i \in \{0, 1\}$, which represents rejection or acceptance of the proposed repayment program. Each bondholder is characterized by a utility function $u(a, \ell, \theta) : \{0, 1\} \times [0, 1] \times [\check{\theta}, \hat{\theta}] \to \mathbb{R}$ in (2).

$$
\begin{aligned}
u(0, \ell, \theta) &= \begin{cases} \delta\nu & \text{if} \quad \ell \geq \ell^* \\ 0 & \text{if} \quad \ell < \ell^* \end{cases} \\
u(1, \ell, \theta) &= \begin{cases} 1 - h - m & \text{if} \quad \ell \geq \ell^* \\ -m & \text{if} \quad \ell < \ell^* \end{cases}
\end{aligned}
\tag{2}
$$

As discussed above, proposal success (assessed in the last stage) will depend on achieving a minimal level of aggregate acceptance ex-post ($\ell > \ell^*$). However, each agent's information set at decision stage is the singleton $\mathcal{I}^3_{inv} = \{\theta\}$. As aggregate acceptance level is unknown while deciding, each agent uses a uniform prior over others' actions (assigns the same probability to each $\ell \in (0, 1)$).

Rejecting agents ($a_i = 0$ in (2)), expect a recovery value of $\delta\nu$ ($\delta \in [0, 1]$) for unit of bond in a successful proposal and 0 otherwise. Agents accepting ($a_i = 1$ in (2)) receive $1 - h$ when negotiation prospers or 0 otherwise, spending in any case participation costs $m > 0$ (for example to acquire the information).

Note that holdouts payments and receipts do not coincide. Here, $1 - \delta$ is a non participating loss which should be interpreted as litigation expenses, the probability of actually receiving that amount and the wait until that happens[10]. $\delta\nu$ is the expected recovery value and as such, should coincide with the bid price of the defaulted bond at the junk market. This situation may portray the bulk of investors for whom expected gains might not compensate litigation costs (low δ). The opposite case is the small group of professional holdouts[11] that buys defaulted debt at secondary market and affords many years of litigation with sovereign at international courts with considerable return. We do not include them in this model, as these group generally weights less than 10% of total

[10] Wright (2011) calibrates restructuring costs in its Nash bargaining model as 3.5% of renegotiated debt, from which 90% falls upon lead investor.

[11] Some of them are Water Street, Elliot Associates, Cerberus, Davidson Kempner, Aurelius Capital.

outstanding (Das et al. 2012), and their behavior does not coincide with the mass of bondholders.

2.3 Full information and multiplicity

2.3.1 Stage 4

Imposing government indifference condition in (1) we obtain a minimal threshold for investors' acceptance $\ell^*(\theta)$ in terms of fundamental. For ℓ above that level, government pays bondholders as agreed ($1-h$ for participants and ν for holdouts) and exits default.

$$\ell^*(\theta) = \frac{\nu - \xi\theta}{\nu - (1-h)} \tag{3}$$

Lemma 1. *Government softens required acceptance in the case of better economic conditions (higher θ), when it commits to a lower repayment (higher h) or when exiting boost (ξ) is higher. On the contrary, $\ell^*(\theta)$ raises when payments to holdouts ν are higher (if $\theta > \frac{1-h}{\xi}$, and lower otherwise).*

Proof: The cases of ξ, θ and h are trivial.

$$\frac{\partial \ell^*(\theta)}{\partial \nu} = \frac{-(1-h)+\xi\theta}{(\nu-(1-h))^2} \geq 0 \text{ for } \theta \geq \frac{1-h}{\xi} \blacksquare$$

Thus given h, the response to increases on holdout payments is not linear. A compromised government position (below $\frac{1-h}{\xi}$) obliges it to reduce $\ell^*(\theta)$ each time ν augments with the purpose of gathering additional acceptance to finance the more expensive holdout payments. On the contrary, high fundamental draws allow the government to increase the threshold reducing the probability of success if it estimates exit might be too expensive.

We can evaluate $\ell^*(\theta)$ in the boundaries of $\ell \in [0,1]$ to identify strict dominance regions in government's strategies as a function of fundamental θ (see figure 2). Thus $\underline{\theta} \equiv \frac{1-h}{\xi}$ and $\bar{\theta} \equiv \frac{\nu}{\xi}$ denote the pair of benchmark values of θ which demands limit levels of engagement, $\ell^*(\underline{\theta}) = 1$ and $\ell^*(\bar{\theta}) = 0$ to exit default. As a consequence, for $\theta \in [\check{\theta}, \underline{\theta})$ default is best response regardless ℓ due to extremely reduced payment capacity, while for $\theta \in [\bar{\theta}, \hat{\theta}]$ abundant assets make exiting default the best strategy even at null acceptance level.

Intuitively, using these limits in the utility function (1), low values of θ request each circulating bond to accept haircut h ($\ell^* = 1$) in order to exit default, and in this case all the assets gain $\xi\theta$ would be applied to comply the program $(1-h)$.

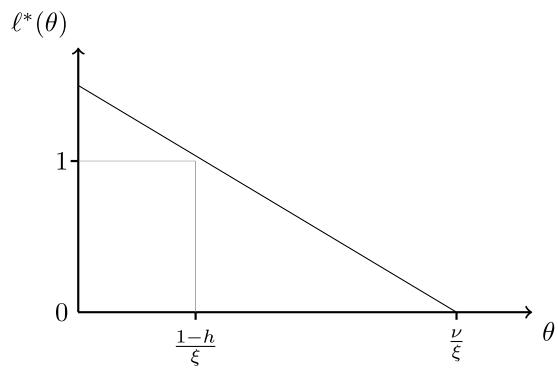


Figure 2: Threshold $\ell^*(\theta)$.

On the contrary, high fundamental draws such as $\bar{\theta}$ can afford every obligation even the holdouts at higher rate ν at which it would apply only the asset gain $\xi\theta$.

2.3.2 Stage 3

This third stage, constitutes a symmetric binary action coordination game. We will use in this section a general version with a continuum of agents. The reader can find a full information two investors illustration in appendix B.

With complete information, each value of θ determines a subgame where investors decide its strategy $a_i(\theta)$ as a best response to others' behavior in the corresponding scenario. The discussion below demonstrates we obtain a pair of equilibrium strategies, depending on parameters $1-h-m$ and $\delta\nu$: $a_i(\theta) = \{(0,0),(0,1),(1,1)\}$ if $\delta\nu < 1-h-m$ and $a_i(\theta) = \{(0,0),(0,0),(0,0)\}$ if $\delta\nu > 1-h-m$, where the elements correspond to contingent scenarios for θ and each parenthesis contains responses to low and high aggregate acceptance (against the threshold ℓ^*) respectively.

For $\theta \in [\check{\theta}, \underline{\theta})$ sovereign continues on default regardless of participation rate ℓ. Participation payoff is strictly below non-participation's : $u(1,\ell,\theta) = -m < u(0,\ell,\theta) = 0$. The strictly dominant strategy is to reject proposal if $m > 0$, for any $\delta\nu$ at both possible scenarios of acceptance (low and high), and as a result $\ell = 0$.

For $\theta \in [\underline{\theta}, \bar{\theta})$ sovereign's decision depends on overall acceptance ℓ. If $\delta\nu < 1-h-m$ agents should all respond accepting when acceptance is low and rejecting otherwise so we can get both $\ell = 1$ or $\ell = 0$. In other words, there are two pure strategy Nash equilibria in this zone: full and zero investor participation, with the first of them Pareto preferable to the second ($1-h-m > 0$). If

$\delta\nu > 1 - h - m$ agents should reject for all ℓ.

For $\theta \in [\bar{\theta}, \hat{\theta}]$ sovereign pays and exits default $\forall \ell \geq 0$, so agents' participation would not risk any loss (in others' decisions). Agent's best response depends on payoff parameters. When $\delta\nu$ is such that $1 - h - m > \delta\nu$, accepting is the payoff dominant strategy, $\ell = 1$ yielding a full participation unique equilibrium. On the contrary, if $1 - h - m < \delta\nu$, rejecting proposal is payoff dominant and $\ell = 0$, in a unique no participation equilibrium. Finally if $1 - h - m = \delta\nu$ agents are indifferent between accepting or rejecting and we obtain a unique mixed strategies equilibrium with $\ell \in [0, 1]$.

Lemma 2. *WLG, we can assume that in equilibrium $\delta\nu < 1 - h - m$ so that holdouts expected payment imposes government a limit when determining haircut level which turns stage 2 into a two equilibria coordination game.*

Proof: If $\delta > 1 - h - m$ there is general rejection with $\ell = 0$, and unless $\theta > \bar{\theta}$, government would remain on default. A participating equilibrium would then be bounded below by off proposal payoff: $\delta\nu \leq 1 - h - m$. For agents to be indifferent, government policy should be to pay $1 - h = \delta\nu + m$, and in this case the game delivers an internal solution ($\ell \in (0, 1)$). That equilibrium, however has zero probability of occurrence, because even for a small ϵ fixing the proposal so that $1 - h + \epsilon - m = \delta\nu$ gets full acceptance. So government's convenience will drive to a full participating equilibrium by setting $1 - h - m > \delta\nu$ ■

A consequence of Lemma 2, is that the incentive to hold out the acceptance in this model results from agents avoiding to lose $-m$ in case proposal fails, and not from an extra expected payment ($\delta\nu < 1-h-m$ is obviously not a preferred payoff).

Summing up, in stage 3, each value of fundamental θ determines a bayesian subgame that solves into unique zero and full participation Nash equilibria outside $(\underline{\theta}, \bar{\theta}]$. However, values of θ inside that range produce multiplicity with both full participating and no participating equilibria.

2.3.3 Stage 1

In this stage government determines optimal haircut h as the value that maximizes expected utility $G^*(h) \in \mathbb{R}$ in θ. Remember that just as the rest of the market, government still does not know exactly fundamental θ value (drawn at stage 2) and rather assumes it takes some value in $[\check{\theta}, \hat{\theta}]$.

In the previous stage we could identify dominance regions in bondholder's strategic behavior. These however, yield multiple equilibria at corner solutions (all bondholders participate and sovereign exists default and no one participates and sovereign remains in default) which is obviously not an interesting result for government's problem.

2.4 Incomplete information

In this section we introduce some noise in investors' information using a global games approach to refine multiplicity.

In this setting, uncertainty about θ remains, but once nature makes a draw from distribution, government observes it directly, while investors only receive a noisy signal of it. Then uncertainty affects both types of agents but incomplete information will only affect investors.

The new features do not modify stage 4 so this section will focus on model performance over stages three to one.

2.4.1 Stage 3

Now investors' information set is $\mathcal{I}^3_{inv} = \{x_i\}$ where $x_i = \theta + \sigma\varepsilon_i$ corresponds to a noisy private realization of fundamental θ with ε_i independent standard normal perturbations and $\sigma > 0$ a scaling parameter. In this setting, a strategy $s_i(x_i)$, for creditor i is a decision rule that maps from the space of signals to that of actions: $\mathbb{R} \to \{0, 1\}$. Accordingly, an equilibrium is a profile of strategies that maximizes each creditor's expected payoff conditional on the information available.

It is important to note that investor's payoffs still depend on the realized value of θ not on his private signal which literature calls common values model. Morris and Shin (2002), propose a general framework to solve such games consisting of a series of sufficient conditions for the payoffs gain function: $u(1, \ell, \theta) - u(0, \ell, \theta)$. For those payoff functions compliant, it is possible to obtain a strategy profile s that conforms a unique Bayesian Nash equilibrium of this game.

Proposition 1. *Let θ^* be defined as in (4).*

$$\theta^*(h) = \left(1 - h + m\frac{\nu - (1-h)}{1 - h - \delta\nu}\right) \tag{4}$$

For any $\tau > 0$, there exists $\bar{\sigma} > 0$ such that for all $\sigma \leq \bar{\sigma}$, if strategy s_i survives iterated deletion of strictly dominated strategies, then $s_i(x_i) = 0$ for all $x_i \leq \theta^* - \tau$, and $s_i(x_i) = 1$ for all $x_i \geq \theta^* + \tau$.

Proof: Appendix C.

A take away from the demonstration of Proposition 1, is threshold $\theta^*(h) \in [\underline{\theta}, \bar{\theta}]$ in (4) which discriminates regions of strict dominance in the game: agents with signals $x_i > \theta^*(h)$ accept proposal and those that observe signals below that level reject it.

Characterization of threshold $\theta^*(h)$

Some algebra on (4) allows us to express threshold $\theta^*(h)$ as a convex combination of government payments to each set of investors (accepting and rejecting proposal) in (5), with the ratio of net costs to benefits of accepting as weightings.

$$\theta^*(h) = \frac{1}{\xi}\left((1-h)(1-\frac{m}{1-h-\delta\nu}) + \nu\frac{m}{1-h-\delta\nu}\right) \tag{5}$$

In this expression, we can see that low values of participation cost m move the threshold away from holdouts payment as they increase the probability of a high acceptance result.

Although $\theta^*(h)$ is determined by the complete set of parameters in the model, not all of them affect it through the same channel. For instance, m and δ contribute directly via agents' payoff function. ξ has an indirect effect on $\theta^*(h)$ by affecting government threshold $\ell^*(\theta)$, and then modifying total acceptance required to shift the state, and the probability of different payoffs with it. Finally haircut h exerts both a direct (through agents decisions) and an indirect (through $\ell^*(\theta)$) effect on the threshold. These relations are the key content of the corollary below.

Corollary 1. $\frac{\partial\theta(h)^*}{\partial\xi} \leq 0$, $\frac{\partial\theta(h)^*}{\partial\nu} \geq 0$, $\frac{\partial\theta(h)^*}{\partial\delta} \geq 0$, $\frac{\partial\theta(h)^*}{\partial} \geq 0$.

Increases in holdout receipts, holdouts payments or participation costs discourage investors acceptance while higher after-restructuring boost (ξ) encourages it.

Proof: Appendix E.

When participation costs m or holdout receipts δ rise, net participation benefit reduces (for higher costs or a better outside option) increasing the probability of rejection (as $\theta^*(h)$ expands). Note that high δ or m values comprise the haircut to the minimum, and even result in complete rejection in the limit, when outside option is competitive or participation is too expensive.

Better after-restructuring conditions (increase ξ) make the program more attractive for the government whom then reduces the participation threshold $\ell^*(\theta)$ to augment its probability of success. This in turn reduces the chance of losing m

for accepting a failed program and then encourages investor participation (by comprising θ^*). In the case of holdouts payments ν, they produce the opposite effect through the same channel, due to the fact that increases in ν force the government to tighten ℓ^*.

As ν converges to $1-h$, $\theta^*(h)$ converges to $\underline{\theta} = \frac{1-h}{\xi}$ its lowest boundary, expanding acceptance zone towards its maximum extension. ν at its lowest boundary, implies lower obligations for the government that reduces $\ell^*(\theta)$ in order to encourage participation to the maximum (as $\frac{\partial \ell^*(\theta)}{\partial \nu} \geq 0$ in $\theta \geq \frac{1-h}{\xi}$). On the contrary, $(1-h)$ converging towards ν pushes θ^* to $\bar{\theta}$ reducing acceptance region to the minimum: in this case there would be little gain from the program and the government must require $\ell^*(\theta) = 1$, decreasing the probability of success.

θ as a function of haircut

Corollary 2. *$\theta^*(h)$ is a U-shaped convex function that minimizes at:*

$$h_{inv} = 1 - \delta\nu - \sqrt{m(1-\delta)\nu}$$

Proof: Appendix E.

So there first is an indirect effect through government's threshold ℓ^*. As h increases, government commits to pay a smaller share of defaulted debt and the relief allows it to reduce minimal acceptance threshold for proposal (3). For investors, that implies a contraction in the probability of losing m (when entering a failed proposal). As a consequence, acceptance region increases by decreasing lower limit $\theta^*(h)$. The direct effect, on the contrary, affects $\theta^*(h)$ through investors' payoff: a higher h, determines a reduction in marginal utility of accepting proposal discouraging investors that in consequence extend non acceptance region (increasing $\theta^*(h)$).

Drawing upon convexity we can optimize $\theta^*(h)$ in h (appendix E). In this unique value which we denominate h_{inv}, the threshold reaches its minimum, implying that accepting region expands most (and the probability of a successful proposal with it) for a given set of parameters.

$$1 - h_{inv} - \delta\nu = \sqrt{m(1-\delta)\nu} \tag{6}$$

This result contrasts with the idea that every sufficiently low h coordinates investors in the full acceptance equilibrium: here, however, there is a trade off for h that originates in strategic behavior. Although bondholders' investment recovery rate increases with a lower haircut (so we would expect higher participation rate), at the same time government situation deteriorates and threshold $\ell^*(\theta)$

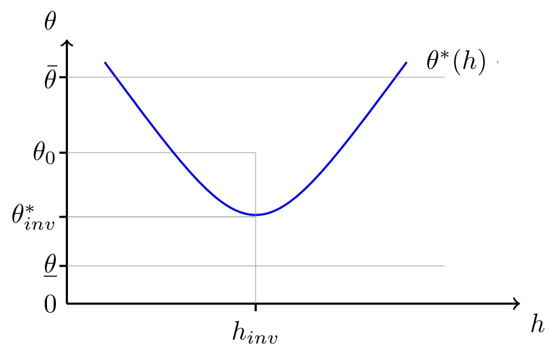


Figure 3: Threshold $\ell^*(\theta)$.

is raised, demanding a higher market support for the program which in turn increases the probability of failure and the lose of m for those that participated (determining a lower participation rate).

That h_{inv} level, however, would not be attainable by the government. For instance, replacing (6) in (3) with $\ell^* = 1$ we obtain the lowest payment capacity required to exit default when haircut is set at h_{inv} level, which we will denote by θ_0.

$$\theta_0 = \frac{1}{\xi}\left(\delta + \sqrt{m(1-\delta)\nu}\right) > \frac{1}{\xi}\left(\delta + 2\sqrt{m(1-\delta)\nu} - 1\right) = \theta^*_{inv} \quad (7)$$

As it shows in (7),[12] had the government set the haircut in h_{inv}, it would enter default for every observed $\theta \in [\check{\theta}, \theta_0]$ which is larger than $[\check{\theta}, \theta_{inv}]$. So at least at this level of haircut, government constraint adds $\theta_{inv} - \theta_0$ to the probability of keeping at default.

Another consequence of convexity in $\theta^*(h)$ is that for any h not equal to h_{inv} there is a pair of values that results in the same $\theta^*(h)$, and as a consequence in the same acceptance level. The effect of this duality in government decision is the main content of Proposition 2.

Proposition 2. *The set of government's available values of h to determine the equilibrium haircut is $h_{gov} \in [h_{inv}, 1 - \delta\nu - m)$.*

Proof: Trivial as government's utility is an increasing function on h■

Observe finally, that the presence of m is critical to determine optimal h. In this model, it is the existence of participation costs which forces both extreme

[12] $\theta_{inv} > \theta_0$ requires $\sqrt{m(1-\delta)\nu} > 1$ which would be false due to $1 \geq \nu > \delta \geq 0$ and m small.

values of h_{gov} to differ from $1-\delta\nu$. So increasing costs demand a lower haircut to keep investors entrance.

2.4.2 Stage 1

Having identified investors' optimal responses to θ, now the government computes the optimal haircut, as the value that maximizes its expected utility in (1) conditional to θ.

$$G^*(h) = \int_{\check{\theta}}^{\theta^*(h)} \theta f(\theta) d\theta + \int_{\theta^*(h)}^{\hat{\theta}} (\theta(1+\xi) - (1-h)) f(\theta) d\theta \tag{8}$$

Proposition 3. *There exists a unique h_{gov} value that solves the model complying parameters' constraints, which maximizes government's expected utility.*

Proof: Appendix F.

We reduce the expression (8) by applying both simple and truncated expectation formulas.

$$G^*(h) = \frac{\hat{\theta}+\check{\theta}}{2} + \xi \left(\frac{\hat{\theta}-\theta^*(h)}{\hat{\theta}-\check{\theta}} \right) \frac{\hat{\theta}+\theta^*(h)}{2} - (1-h) \frac{\hat{\theta}-\theta^*(h)}{\hat{\theta}-\check{\theta}} \tag{9}$$

Differentiating and solving for h, we obtain h_{gov}, the haircut level that maximizes government utility.

Proposition 4. *h_{gov} solves:*

$$-m^2(\nu-\delta)(\nu-(1-h_{gov})) + (\hat{\theta}\xi - (1-h_{gov}))(1-h_{gov}-\delta\nu)^3 = 0$$

Proof: Appendix G.

Haircut as a function of model parameters

Corollary 3. *Under certain conditions (sufficiently high values of $\hat{\theta}$) h_{gov} is: decreasing in ν, δ and increasing in ξ and $\hat{\theta}$.*

Proof: Appendix H.

So for sufficiently high values of $\hat{\theta}$ optimal haircut is decreasing in holdouts receipts, holdouts payments and participation costs, while it is increasing in post-exit boom and the upper bound of distribution.

Haircut reduces with holdouts receipts δ and participation costs m as the government tries to compensate bondholders whose outside option and costs increased to secure its participation. In the limit, zero costs allow the government to situate haircut at the highest possible value $h_{gov} = 1 - \delta$.

Holdouts payments ν, also exert a negative effect on haircut through both government's and investor's strategic decisions. Unlike the previous case, increases in ν do not affect investors utilities but the threshold $\theta^*(h)$: higher payments to holdouts reduce the probability of success which expands the rejection region. In addition, a higher ν erodes government net assets position which then boosts required participation ratio. This decision in turn discourages bondholders participation, so once again government compensates by reducing h.

Haircut increases with post exit boom, via again both investors' and government's channels. In the first case, the exit boom ξ affects the threshold by increasing the acceptance zone once government reduced $\ell^*(\theta)$ with the expected better results after the negotiation. Besides, the increase in post-negotiation boom encourages government to reduce haircut in order to secure agreement and obtain a higher utility.

Implicit derivation on (4) unveils that when uncertainty about the state expands the probability of higher draws of states increases, fading participation rate relevance. The government profits on this situation by increasing the haircut which yields a higher utility.

It is not surprising that lower limit $\check{\theta}$ does not appear in h_{gov}. This comes as a consequence that in $[\check{\theta}, \theta^*)$ the solution is to redefault regardless the value of h, so low values of θ are not determinant in defining the optimal h.

As detailed in corolary 3 there is a condition on parameters for the above relations to hold. Indeed, in case $\hat{\theta}$ situates below $\underline{\theta} + \frac{1-h-\delta}{3\xi} - \frac{m^2(\nu-\delta)}{(1-h-\delta\nu)^2}$, the renegotiation is not a gain and government would move haircut in order to keep on default.

3 Model assessment

This section dedicates to the appraisal of the model using simulated results with calibrated parameters. In the first part, there is a comparison between coordination haircut and the haircut widely used in restructuring literature: Nash bargaining. We find that in most cases, Nash bargaining haircut situates above coordination one. In the second part, we propose a procedure to assess the cost to the government of changing from bank financing towards market financing, which we will call coordination costs. We find that these costs can reach a 10% level which accounts for a third of the average difference between both financing types discussed previously.

3.1 Coordination vs Nash bargaining haircut

In this section, we compare our results with the ones obtained through the equilibrium solution of Nash bargaining.[13] Yue (2010) argues that the result of the restructuring process affects the recovery path and so there is a value in endogenizating the haircut level, against previous Aguiar and Gopinath (2006) and Arellano (2008) where exiting the state of default supposes a haircut of 100% or an exogenous random value.

Nash bargaining, although a helpful tool to find equilibria en negotiation processes relies critically on participants' bargaining power, which is not observable and has to be calibrated to fit some feature of the data. In Yue (2010) movements in bargaining power of about 18% produce variations of 3% in average debt to output ration, 13% in average recovery rates and meaningful changes in correlations. The model in this paper provides an endogenous determination of such bargaining power in terms of tractable parameters in the model: economy expected recovery rate, participation costs, secondary market price.

To compare both results, we estimated Nash bargaining equilibrium in term of the parameters of this model. We estimated haircut as the value that maximizes expected Nash product in θ. We assume government has a bargaining power of α and investors $1-\alpha$. Government's payoff function in (1) provides the gain of restructuring and the outside option θ available. In the case of bondholders, the outside option to participate yields a payoff of $\delta\nu$ as no one agent can change the overall restructuring result.

Using this setting we obtain the haircut for Nash bargaining in (10) (Appendix J).

$$h = \frac{\alpha\ell(1-m-\delta\nu) - (1-\alpha)(\xi\bar{\theta} - \nu - \ell(1-\nu))}{\ell} \tag{10}$$

Parameters in simulations are summed up in table 2. The value of ν was set following average recovery value in Cruces and Trebesch database plus the weighted average of holdout premium in Fridson and Gao (2002) and Altman and Eberhart. δ value was selected in order to target market price $\nu\delta = 0.3$ as in the weighted average presented in Moody's investors service data report (2017) for 30-day post-default price or distressed exchange trading price. The value of ξ was set to 0.065 in what we understand would be a conservative value if we consider Das (2012) average economy's recovery rates after restructuring. m

[13] See for instance Yue (2010), Asonuma (2012), Arellano (2008), Bai, Zhang (2012)

Parameters	
ν	0.835
δ	0.375
ξ	0.065
m	0.006
α	0.7

Table 2: Model parameters in simulations

is a non observable participating cost it was calibrated to be less than 1% in total investment $m = 0.006$. Government's bargaining power α was set in 0.7 following Yue (2011).

In figure 4 we plotted both results, for the previous parametrization in the case of h_{gov} and the referred for h_{NB}. Using a bargaining power of 70% for the government as in Yue (2011) Nash bargaining benchmark outperforms our proposed model in most cases by a significant amount evidencing that in this setting, government holds an extraordinary power (even when investor's outside option is $\delta\nu$ if program fails, and not 0). The wedge is robust to α levels around the calibrated parameters[14]. From another perspective, the introduction of coordination amongst bondholders surprisingly has a *de facto* weakening effect on government bargaining power even when the secondary market atomizes individual bondholder participation during negotiation.

Wrapping up, a more detailed micro founded model signals there is a loss of bargaining power that derives from the bondholders playing a bargaining game. These loss can be traced in observable economic parameters avoiding the calibration of an appropriate value for bargaining power α.

3.2 Nash bargaining and coordination costs

In this section we try to reach a measure for coordination costs. We start by measuring the haircut in a scenario in which government has all the bargaining power, but it confronts a unique investor instead of a continuous of unorganized agents. These features might be captured by a Nash bargaining haircut with government's full bargaining power, which entails $\alpha = 1$. Using this parameter in (J) we get:

$$h_{NB,\alpha=1} = 1 - \delta\nu - m \tag{11}$$

Which coincides with the net outside option value for investors and the upper bound for h according to lemma 2. We can then calculate coordination costs

[14] Das (2012) reports a maximum real recovery rate of 4.8% in observed cases.

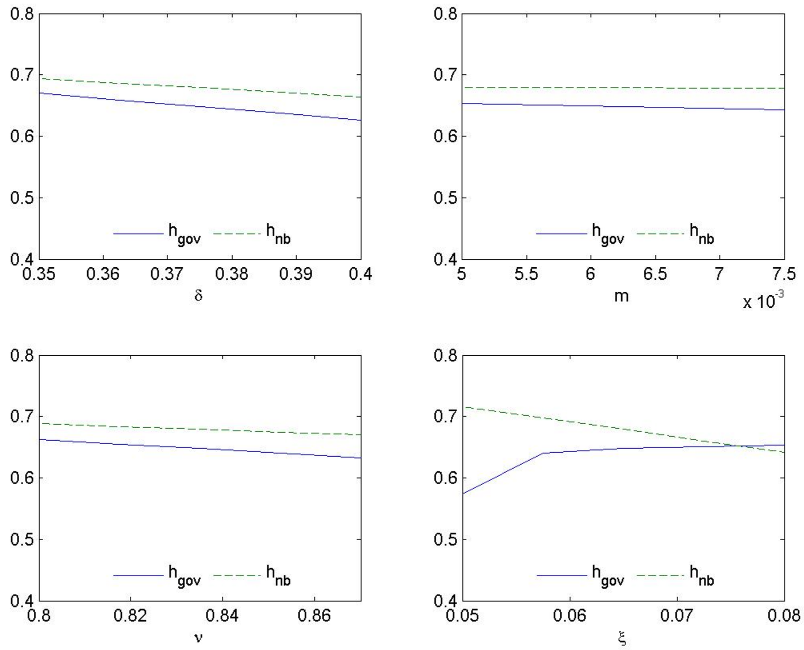


Figure 4: Comparison of h_{gov} against h_{NB}, using $\delta = 0.375$, $\nu = 0.835$, $\xi = 0.065$, $m = 0.006$, $\alpha = 0.70$, $\ell = 1$ in general and haircuts as a function of: (a) δ, with $\delta \in [0.35, 0.40]$; (b) m, with $m \in [0.005, 0.0075]$; (c) ν, with $\nu \in [0.80, 0.87]$; (d) ξ, with $\xi \in [0.05, 0.08]$

has the difference in both haircuts:

$$coordination\, costs = h_{NB,\alpha} - h_{gov} \tag{12}$$

Using previous calibration, we simulate those coordination costs (figure 5) and we observe that they can situate in 3-10% range, where the critical parameter is the fundamental recovery value ξ.

It is important to note that this results are in line with the fact that the dis-intermediation in sovereign debt financing determined a reduction in haircut. In other terms, in this new market setting, the government has a cost derived from the fact that it does not bargain with its counterparty and instead it has to target higher order beliefs with relased haircut (all agents has to think that the rest will participate as it is an appropiate proposal).

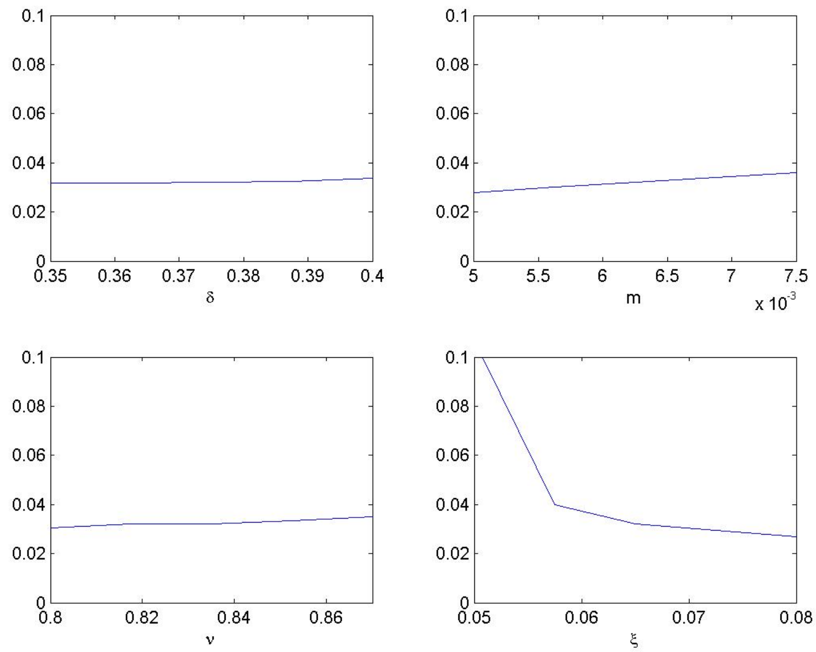


Figure 5: Cooordination costs as a function of model parameters, using $\delta = 0.375$, $\nu = 0.835$, $\xi = 0.065$, $m = 0.006$, $\alpha = 0.70$, $\ell = 1$ in general and haircuts as a function of: (a) δ, with $\delta \in [0.35, 0.40]$; (b) m, with $m \in [0.005, 0.0075]$; (c) ν, with $\nu \in [0.80, 0.87]$; (d) ξ, with $\xi \in [0.05, 0.08]$

4 Conclusions

After a strong process of financial disintermediation starting in 1980, negotiating an exit from defaulted debt is a complex process; it requires to align the decisions of a numerous set of investors (many thousands) in a relative short period of time with limited information regarding the progress accomplished. Final result relies on the aggregated behavior of that mass of spread and mostly unconnected agents elaborating their strategy simultaneously. Surprisingly (as in bargaining literature this atomized set of bondholders might lose every bargaining power), we find that terms and outcomes has leaned towards investors, with shorter negotiations and lower haircuts.

Game theory applied to this background entails backward induction in a coordination scheme that introduces multiple equilibria. These particularly occur over non limit economic conditions, where internal non trivial solutions seem most plausible. We apply then global games to solve multiplicity, requiring amongst other things allocating some costs to participation and noise in investors' information sets.

Maximizing government's utility, we endogenize the haircut which is set in terms of model parameters, fundamental distribution and a minimal threshold on it for bondholders' participation. Working with homogeneous bondholders yields full participation which prevents us to rely on contract coordination devices (special majorities, exit consents, etc).

We find, convexity on the haircut when determining investors strategic decisions. Holdouts receipts and participation costs, fix both lower and upper bounds to government concession.

We embed a Nash bargaining model into ours, and find that pure Nash bargaining result obtain higher haircut values for simulated exercises in calibrated parameters. We conclude that by introducing coordination, we set a *de facto* restriction on government's bargaining power, as it is forced to compress its proposal in order to coordinate bondholders towards the participation option (trying to align second order beliefs). Using simulated results, coordination costs 2-10% in terms of proposed haircut, which explains a significant portion of observed difference between the haircuts set during restructurings with banks against bondholders in secondary markets.

This model contents two main limitations. In the first place, due to symmetric-investor specification, we cannot project participation rates (here it is zero or hundred percent), neither explain holdout historical behaviour. In the second place, optimal haircut is finally unresponsive of default costs, which seems an undesirable feature. Further work should study the effects of heterogeneous investors and other informational settings.

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Appendix

A Haircuts, some empirical facts

In this section we analyze empirical facts from haircuts using the base of Cruces & Trebsech (2014 updated). Our variable of interest is the restructuring haircut (a measure of the total concession from debtors to creditors). The preferred measure is the one proposed in Sturzenegger & Zettelmeyer (2006 and 2008) in which haircut summarizes the present value of investors' losses:

$$H^i_{SZ} = 1 - \frac{\text{Present value of New debt}(r^i_t)}{\text{Present value of old debt}(r^i_t)} \tag{13}$$

Where r^i_t is the yield prevailing at the time of the restructuring, which is considered a good proxy of debtor's default risk after the restructure.[15]

	Observations	Mean	SD	Mean	Max
SZ Haircut	187	.40	.28	-.098	.97
By type of creditor					
Bank debt restructuring	165	.37	.28	-.098	.97
Bond debt restructuring	22	.37	.22	.04	.76
By era					
1978–1989	99	.25	.19	-.098	.93
1990–1997	48	.51	.28	.03	.92
1998–2013	40	.52	.32	-.08	.97

Table 3: Haircut summary. Haircut measured as in Sturzenegger Zettelmeyer (2006) by type of creditor and era. Source: Cruces and Trebsesch (2013) updated with 2014 new data.

Table 3, presents a summary. As it can be noted, there were 187 cases of restructurings during 1978-2013 with a mean value of haircut of 40%. A break up by type of creditor shows that there are only 22 cases of bond restructuring, although the average haircut does not seem to differ with its characteristic. Finally, the period when the process took place seems to be an important element. In fact, before 1990 haircuts where set around 25% while after that date they double up to 50% on average. These results suggest that the year in which the process took place will be a variable to control for in order to better understand haircut determinants.

[15] Cruces and Trebesch (2013)

In Table 4, splitting the sample by era, we compare haircut means by type of creditor. With the whole sample, consistent with previous result, both groups bank and bond restructuring show no significant differences on mean averages. When we split the sample using the variable era, we find that most bond restructuring situate at recent years (19 out of 22 cases in 1998-2020). Within this period there almost a 30% difference in the haircut negotiated in restructuring, which results a significant difference according to ttest.

	Bank	Bond	Difference	p-value
All sample	0.37	0.38	0.00	0.96
N	165	22		
1998-2013	0.66	0.39	0.27	0.01
N	19	21		

Table 4: Mean haircut by type of creditor and era

Table 5 presents regression analysis results for the logarithm of haircut when using bonds against bank loans controlling by era, and others (logarithms of GDP pc and GDP). As the reader can note, once controlled by year, creditor nature results significant at least at 5% level and with a persistent negative sign. A little algebra on results indicates that the ratio of haircuts in bonds to banks situates at 43% and 52% in models (1) and (2) respectively.

Haircut	(1)	(2)
Bond dummy	-0.84***	-0.64*
	(0.25)	(0.25)
Constant	*Yes*	*Yes*
Decade	*Yes*	*Yes*
Other controls	*No*	*Yes*
N	180	162
adj. R^2	0.16	0.29

Table 5: Model estimates for the logarithm of Haircut. Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Other controls: GDP and GDP pc in logarithmic units.

B Two investors illustration

Consider a two investors game where each of them has full information about economic fundamental θ. The game is sketched in the tables below, with investor one's strategies presented in rows and investor two's in columns. Payoffs are ordered in rows inside each cell respectively.

Government proposes a concession program whose result depends on observed fundamental θ. This variable in time, determines payoff matrix and finally game equilibria as detailed below.

If $\theta < \underline{\theta}$ then $\ell^*(\theta) > 1$ in (3), meaning that the program fails even at 100% acceptance. In this case, there are no benefits for accepting investors: acceptance has a cost of $-m$ while rejection costs 0. Payoffs matrix is Table 6 that shows weakly dominance in the unique Nash equilibrium in pure strategies, $s = \{s_1, s_2\} = \{Reject, Reject\}$.

Investor 1 \ Investor 2	Accepts	Rejects
Accepts	$-m$ $-m$	$-m$ $\underline{0}$
Rejects	$\underline{0}$ $-m$	$\underline{0}$ $\underline{0}$

Table 6: Payoffs matrix $\theta \in [\check{\theta}, \underline{\theta})$

Draws of $\theta \in [\bar{\theta}, \hat{\theta}]$ produce $\ell^*(\theta) \leq 0$, unconditional restructuring. The game pays $1-h-m$ to accepting investors and δ to rejecting ones (assume here that $\delta < 1-h-m$). This game in Table 7, solves into a unique pure strategies Nash equilibrium, with weakly dominance in $s = \{s_1, s_2\} = \{Accept, Accept\}$.

Investor 1 \ Investor 2	Accepts	Rejects
Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$\underline{1-h-m}$ $\delta\nu$
Rejects	$\delta\nu$ $\underline{1-h-m}$	$\delta\nu$ $\delta\nu$

Table 7: Payoffs matrix $\theta \in [\bar{\theta}, \hat{\theta}]$

Finally, Table 8 portrays $\theta \in [\underline{\theta}, \bar{\theta})$ and then $\ell(\theta)^* \in [0, 1]$. Suppose we can identify a θ_0 value under which $\ell^*(\theta) > 0.5$: government requires both investors accepting to exit default. In this case (left box in the table) we get two equilibria in strict dominant strategies with mirror behavior. Above that level of θ government exits default even if only one investor participates (right box in the table). Now acceptance is a weakly dominant strategy and we get an all participating equilibrium.

Remember a strategy is an action plan for every contingency. We then have:

$$a_i = \{(Reject, Reject), (Accept, Reject), (Accept, Accept), (Accept, Accept)\}$$

Investor 1 \ Investor 2	Accepts	Rejects
Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$-m$ 0
Rejects	0 $-m$	$\underline{0}$ $\underline{0}$

θ low $\rightarrow \ell^* > 0.5$

Investor 1 \ Investor 2	Accepts	Rejects
Accepts	$\underline{1-h-m}$ $\underline{1-h-m}$	$\underline{1-h-m}$ $\delta\nu$
Rejects	$\delta\nu$ $\underline{1-h-m}$	$\delta\nu$ $\delta\nu$

θ high $\rightarrow \ell^* = 0.5$

Table 8: Payoffs matrix $\theta \in [\underline{\theta}, \theta]$

Where elements represent contingent fundamental subsets $\{[\check{\theta}, \underline{\theta}), [\underline{\theta}, \theta_0), (\theta_0, \bar{\theta}], [\bar{\theta}, \hat{\theta}]\}$, and inside each parenthesis we portray optimal responses to others' accept and reject decisions respectively.

The multiplicity originates at the second region, where we can get indistinctly an all accepting and an all rejecting equilibrium.

C Equilibrium uniqueness

From (2) we construct in (14) the action gain function $\pi(\ell, \theta) : [0, 1] \times \mathbb{R}^+ \rightarrow \mathbb{R}$ as $\pi(\ell, \theta) = u(1, \ell, \theta) - u(0, \ell, \theta)$.

$$\pi(\ell, \theta) \equiv \begin{cases} 1 - h - \delta\nu - m & \text{if} \quad \ell \geq \ell^* \\ \\ -m & \text{if} \quad \ell < \ell^* \end{cases} \tag{14}$$

According to (14), agents make a $1 - h - \delta\nu - m$ net profit for accepting the agreement when it proceeds, and a $-m$ net loss for accepting it when it fails. Now we demonstrate equation (14) compliance with each condition:

- **C1: Action monotonicity: incentive to choose action $a = 1$ is increasing in ℓ.**
  $\pi(l, \theta)$ is a step function in ℓ discontinuous at $\ell = \ell^*(\theta)$. If $1-h-\delta\nu-m > -m$ then $\pi(\ell^{*+}, \theta) = 1 - h - \delta\nu - m > \pi(\ell^{*-}, \theta) = -m$ and function is increasing in other players' actions, implying strategic complementarity in the game. If $1 - h - \delta\nu - m < -m$, $\pi(\ell, \theta)$ is decreasing in ℓ: rejecting government proposal is a strictly dominant strategy and the game turns into a strategic substitutes structure. Coordination equilibrium requires $1 - h - \delta\nu - m > 0 > -m$ to ensure action monotonicity.

- **C2: State monotonicity: the incentive to choose $a = 1$ is non decreasing in fundamental θ.**
  $\ell^*_\theta(\theta) < 0$ in (3), implies that increases in θ reduce the lower bound of

$\pi(\ell^+, \theta)$, expanding the region where action $a = 1$ is a dominant strategy (given C1 compliance: $\pi(\ell^{*+}, \theta) > \pi(\ell^{*-}, \theta)$). Which can be interpreted as: worse economic conditions weaken government requirement of minimal acceptance which increases probability of a successful process.

- **C3: Strict Laplacian state monotonicity: ensures there is a unique crossing for a player with Laplacian beliefs.** Intuitively, it derives from the fact that players in the game assume a uniform distribution to the proportion ℓ of other players choosing action $a = 1$.

$$\int_{\ell=0}^{\ell=1} \pi(\ell, \theta) d\ell = \int_{\ell=0}^{\ell=\ell^*(\theta)} -m \, d\ell + \int_{\ell=\ell^*(\theta)}^{\ell=1} 1 - h - \delta\nu - m \, d\ell = 0 \quad (15)$$

Equation (15) can be expressed as a linear function on θ with a unique solution in θ^* (appendix D).

$$\theta^* = \frac{1}{\xi}\left(1 - h + m\frac{\nu - (1-h)}{1 - h - \delta\nu}\right) \quad (16)$$

- **C4: Uniform limit dominance:** There exists a pair of values $\{\theta_0, \theta_1\} \in \mathbb{R}$, and $\varepsilon \in \mathbb{R}_{++}$, such that [1] $\pi(\ell, \theta) \leq -\varepsilon$ for all $\ell \in [0, 1]$ and $\theta \leq \theta_0$; and [2] there exists θ_1 such that $\pi(\ell, \theta) > \varepsilon$ for all $\ell \in [0, 1]$ and $\theta \geq \theta_1$.
From (3) we know that $\ell^*(\theta)$ is a linear function with value 1 at $\underline{\theta} = \frac{1-h}{\xi}$ (figure 1). Using $\ell^*_\theta(\theta) < 0$ in lemma 1 we can define $\theta_0 = \frac{1-h}{\xi} - \varepsilon$, such that $\ell^*(\underline{\theta}) = 1 + \varsigma$ (for ς very small) and as a consequence $\pi(\ell, \theta) = -m$ for all $\ell \in [0, 1]$ and every $\theta \leq \theta_0$.
The analogue demonstration can be done for [2], taking in this case $\theta_1 = \frac{\nu}{\xi} + \varepsilon$.

- **C5: Continuity:** $\int_{\ell=0}^{\ell=1} g(\ell)\pi(\ell, x) d\ell$ **is continuous with respect to signal x and density g.**
$\pi(\ell, x)$ presents only one point of discontinuity at $\ell = \ell^*$. So for a continuous density $g(\ell)$, discontinuity acquires zero mass, then integral is weakly continuous.

- **C6: Finite expectations of signals:** $\int_{z=0}^{z=+\infty} z f(z) dz$ **is well defined for integration.**
With $z = \frac{x-\theta}{\sigma}$. $f(z)$ defined as a continuous density function with $\int_{z=0}^{z=1} f(z) < +\infty$.

As payoff gain function complies with conditions C1-C6, Proposition 1 following states that the game can be solved to deliver a unique equilibrium in terms of fundamental θ. (Morris and Shin (2002))

D Laplacian state monotonicity proof

There is a single cross on θ^* for action gain function π.

$$\int_{\ell=0}^{\ell=1} \pi(\theta,\ell)d\ell = \int_{\ell=0}^{\ell=\ell^*(h)} -m\, d\ell + \int_{\ell=\ell^*(h)}^{\ell=1} 1-h-\delta\nu-m\, d\ell = 0$$

$$-m\ell^*(h) + (1-h-\delta\nu-m)(1-\ell^*(h)) = 0$$

Now we replace ℓ^* for the expression in (3).

$$(1-h-\delta\nu-m) - (1-h-\delta)\frac{\nu-\theta\xi}{\nu-(1-h)} = 0$$

$$(1-h-\delta\nu-m)(\nu-(1-h)) - (1-h-\delta)(\nu-\theta\xi) = 0$$

$$\theta = \frac{-(1-h-\delta\nu-m)(\nu-(1-h)) + (1-h-\delta)\nu}{(1-h-\delta)\xi}$$

$$\theta^* = \frac{1}{\xi}\left(1-h+m\frac{\nu-(1-h)}{1-h-\delta\nu}\right)$$

E Analysis of threshold θ^* partial derivatives

To determine (17), (18) and(20) I use $\nu > 1-h \geq \delta$, $m > 0$ and $\xi \leq 1$:

$$\frac{\partial\theta^*(h)}{\partial m} = \frac{1}{\xi}\left(\frac{\nu-(1-h)}{1-h-\delta\nu}\right) \geq 0 \tag{17}$$

$$\frac{\partial\theta^*(h)}{\partial\xi} = -\frac{1}{\xi^2}\left(1-h+\frac{m(\nu-(1-h))}{1-h-\delta\nu}\right) \leq 0 \tag{18}$$

$$\frac{\partial\theta^*(h)}{\partial\delta} = \frac{1}{\xi}\left(\frac{m(\nu-(1-h))}{(1-h-\delta\nu)^2}\right) \geq 0 \tag{19}$$

$$\frac{\partial\theta^*(h)}{\partial\nu} = \frac{1}{\xi}\left(m\frac{(1-\delta)(1-h)}{(1-h-\delta\nu)^2}\right) \geq 0 \tag{20}$$

$$\frac{\partial \theta^*(h)}{\partial h} = \frac{1}{\xi}\left(-1 + \frac{m(1-\delta)\nu}{(1-h-\delta\nu)^2}\right) \tag{21}$$

Convexity of $\theta^*(h)$ in h allows us to find optimal value h_{inv} in (22) by equating its derivative to zero:

$$1 - h_{inv} - \delta\nu = \sqrt{m(1-\delta)\nu} \tag{22}$$

F Concavity

Taking the second derivative in (31) respect to h:

$$\frac{\partial^2 G^*(h)}{\partial^2 h} = -\frac{\partial^2 \theta^*(h)}{\partial^2 h}\frac{\hat{\theta}+\theta^*(h)}{2(\hat{\theta}-\breve{\theta})}\xi - \left(\frac{\partial \theta^*(h)}{\partial h}\right)^2 \frac{\xi}{2(\hat{\theta}-\breve{\theta})} + \frac{\partial^2 \theta^*(h)}{\partial^2 h}\frac{\hat{\theta}-\theta^*(h)}{2(\hat{\theta}-\breve{\theta})}\xi$$
$$- \left(\frac{\partial \theta^*(h)}{\partial h}\right)^2 \frac{\xi}{2(\hat{\theta}-\breve{\theta})} - \frac{\partial \theta^*(h)}{\partial h}\frac{1}{\hat{\theta}-\breve{\theta}} + \frac{\partial^2 \theta^*(h)}{\partial^2 h}\frac{1-h}{\hat{\theta}-\breve{\theta}} - \frac{\partial \theta^*(h)}{\partial h}\frac{1}{\hat{\theta}-\breve{\theta}} \tag{23}$$

$$\frac{\partial^2 G^*(h)}{\partial^2 h} = \frac{\partial^2 \theta^*(h)}{\partial^2 h}\left(-\frac{\theta^*(h)}{\hat{\theta}-\breve{\theta}}\xi + \frac{1-h}{\hat{\theta}-\breve{\theta}}\right) - \frac{\partial \theta^*(h)}{\partial h}\frac{2}{\hat{\theta}-\breve{\theta}} - \left(\frac{\partial \theta^*(h)}{\partial h}\right)^2 \frac{\xi}{\hat{\theta}-\breve{\theta}} \tag{24}$$

$$\frac{\partial^2 G^*(h)}{\partial^2 h} = \frac{1}{\hat{\theta}-\breve{\theta}}\left[\frac{\partial^2 \theta^*(h)}{\partial h}\left(-\theta^*(h)\xi + 1 - h\right) - \frac{\partial \theta^*(h)}{\partial h}2 - \left(\frac{\partial \theta^*(h)}{\partial h}\right)^2 \xi\right] \tag{25}$$

$$\frac{\partial^2 G^*(h)}{\partial^2 h} = \underbrace{\frac{1}{\hat{\theta}-\breve{\theta}}}_{>0}\left[\underbrace{\frac{\partial^2 \theta^*(h)}{\partial h}}_{>0}\left(\underbrace{-m\frac{\nu-(1-h)}{1-h-\delta}}_{<0}\right) - \underbrace{\frac{\partial \theta^*(h)}{\partial h}}_{>0}2 - \left(\frac{\partial \theta^*(h)}{\partial h}\right)^2 \xi\right] < 0 \tag{26}$$

For proposition 2, government will set $h \in [h_{inv}, 1]$ where $\frac{\partial \theta^*(h)}{\partial h} > 0$. Then $G^*(h)$ is concave inside the objective region.

Second derivative of $G^*(h)$ is negative implying the function has a maximum which is global, inside the region where parameter constraints hold, $1 \geq \nu > 1-h > \delta\nu \geq 0$, $1-h-m > \delta\nu$, $0 < \xi \leq 1$, $0 < m < 1$.

G Government optimal haircut

We want to find $h \in [0,1]$ that maximizes government's expected utility $G^*(h) = E_\theta[G(h)]$, where $\theta \sim U[\check{\theta}, \hat{\theta}]$.

$$h = argmax_{h\in[0,1]}\{G^*(h)\} \tag{27}$$

Government expected utility is presented in (28). Note that for values of θ above θ^*, all agents accept proposal so that $l = 1$.

$$G^*(h) = \int_{\check{\theta}}^{\theta^*} \theta f(\theta) d\theta + \int_{\theta^*}^{\hat{\theta}} (\theta(1+\xi) - (1-h)) f(\theta) d\theta \tag{28}$$

Rearranging terms in (28):

$$G^*(h) = \underbrace{\int_{\check{\theta}}^{\hat{\theta}} \theta f(\theta) d\theta}_{E[\theta]} + \xi \underbrace{\int_{\theta^*}^{\hat{\theta}} \theta f(\theta) d\theta}_{(1-F(\theta^*))E[\theta|\theta>\theta^*]} - (1-h) \underbrace{\int_{\theta^*}^{\hat{\theta}} f(\theta) d\theta}_{1-F(\theta^*)} \tag{29}$$

Now the truncated expectation in this context:

$$E[\theta|\theta > \theta^*] = \frac{\int_{\theta^*}^{\hat{\theta}} \theta f(\theta) d\theta}{1-F(\theta^*)} = \frac{\int_{\theta^*}^{\hat{\theta}} \theta \frac{1}{\hat{\theta}-\check{\theta}} d\theta}{\frac{\hat{\theta}-\theta^*}{\hat{\theta}-\check{\theta}}} = \frac{\theta^2 \,|_{\theta^*}^{\hat{\theta}}}{2(\hat{\theta}-\theta^*)} = \\ = \frac{\hat{\theta}^2 - \theta^{*2}}{2(\hat{\theta}-\theta^*)} = \frac{(\hat{\theta}-\theta^*)(\hat{\theta}+\theta^*)}{2(\hat{\theta}-\theta^*)} = \frac{\hat{\theta}+\theta^*}{2} \tag{30}$$

Back to (29):

$$G^*(h) = \frac{\hat{\theta}+\check{\theta}}{2} + \xi \left(\frac{\hat{\theta}-\theta^*(h)}{\hat{\theta}-\check{\theta}} \right) \frac{\hat{\theta}+\theta^*(h)}{2} - (1-h)\frac{\hat{\theta}-\theta^*(h)}{\hat{\theta}-\check{\theta}} \tag{31}$$

Taking first derivative in h:

$$-\frac{\partial\theta^*(h)}{\partial h}\frac{\hat{\theta}+\theta^*(h)}{2(\hat{\theta}-\check{\theta})}\xi + \frac{\partial\theta^*(h)}{\partial h}\frac{\hat{\theta}-\theta^*(h)}{2(\hat{\theta}-\check{\theta})}\xi \\ + \frac{\hat{\theta}-\theta^*(h)}{\hat{\theta}-\check{\theta}} + \frac{\partial\theta^*(h)}{\partial h}\frac{1-h}{\hat{\theta}-\check{\theta}} = 0 \tag{32}$$

$$\frac{\partial\theta^*(h)}{\partial h}\left(-\xi\frac{\hat{\theta}+\theta^*(h)}{2}+\xi\frac{\hat{\theta}-\theta^*(h)}{2}+1-h\right)+\hat{\theta}-\theta^*(h)=0 \tag{33}$$

$$\frac{\partial\theta^*(h)}{\partial h}\left(-\xi\theta^*(h)+1-h\right)+\hat{\theta}-\theta^*(h)=0 \tag{34}$$

$$\frac{1}{\xi}\left(-1+\frac{m(\nu-\delta)}{(1-h-\delta)^2}\right)\left(-m\frac{\nu-(1-h)}{1-h-\delta}\right) + \hat{\theta}-\frac{1}{\xi}\left(1-h+m\frac{\nu-(1-h)}{1-h-\delta}\right)=0 \tag{35}$$

So the optimal haircut h_{gob} is the h that solves the equation in (36).

$$\frac{-m^2(\nu-\delta)(\nu-(1-h_{gob}))}{(1-h_{gob}-\delta)^3}+\hat{\theta}\xi-(1-h_{gob})=0 \tag{36}$$

H Analysis of h_{gob} partial derivatives

To analyze partial effects of model parameters on optimal haircut over $[\check{\theta},\hat{\theta}]$, we will use implicit function theorem.

For the rest of the section, we applied the following constraints on parameters: $m<1-h-\delta$, $\nu>1-h$, $\underline{\theta}<\bar{\theta}<\hat{\theta}\rightarrow 1-h<\nu<\xi\hat{\theta}$.

Holdouts receipts

$$\frac{\partial h_{gob}}{\partial\delta}=m^2\nu-m^2(1-h)-m^2(\nu-\delta)\frac{dh}{d\delta}+3\hat{\theta}\xi(1-h-\delta)^2(-\frac{dh}{d\delta}-1) - 3(1-h-\delta)^2(1-h)(-\frac{dh}{d\delta}-1)+(1-h-\delta)^3\frac{dh}{d\delta}=0 \tag{37}$$

$$(m^2(\nu-\delta)+(3(\hat{\theta}\xi-(1-h))+1-h-\delta)(1-h-\delta)^2\frac{dh}{d\delta}=m^2(\nu-(1-h)) - 3(\hat{\theta}\xi-(1-h))(1-h-\delta)^2+(1-h-\delta)^3 \tag{38}$$

$$\frac{dh}{d\delta} = \frac{-m^2 \frac{\nu-(1-h)}{(1-h-\delta)^2} + 3\xi(\hat{\theta} - \underline{\theta})}{-m^2 \frac{\nu-\delta}{(1-h-\delta)^2} + 1 - h - \delta - 3\xi(\hat{\theta} - \underline{\theta})} < 0 \tag{39}$$

Post restructuring bounce

$$\begin{aligned} &- m^2(\nu - \delta)\frac{dh}{d\xi} - 3\hat{\theta}(1-\xi)(1-h-\delta)^2\frac{dh}{d\xi} + \hat{\theta}(1-h-\delta)^3 \\ &\qquad + \frac{dh}{d\xi}(1-h-\delta)^3 + 3(1-h)(1-h-\delta)^2\frac{dh}{d\xi} = 0 \end{aligned} \tag{40}$$

$$\frac{dh}{d\xi} = -\frac{\hat{\theta}(1-h-\delta)^3}{-m^2(\nu-\delta) + (1-h-\delta-3(\hat{\theta}\xi-(1-h)))(1-h-\delta)^2} \tag{41}$$

$$\frac{dh}{d\xi} = -\frac{\hat{\theta}(1-h-\delta)}{-m^2 \frac{\nu-\delta}{(1-h-\delta)^2} + 1 - h - \delta - 3\xi(\hat{\theta} - \underline{\theta})} > 0 \tag{42}$$

Limit $\hat{\theta}$

$$\begin{aligned} &- m^2(\nu - \delta)\frac{dh}{d\hat{\theta}} + (1-\xi)(1-h-\delta)^3 - 3\hat{\theta}\xi(1-h-\delta)^2\frac{dh}{d\check{\theta}} + \frac{dh}{d\hat{\theta}}(1-h-\delta)^3 \\ &\qquad + 3(1-h-\delta)^2(1-h)\frac{dh}{d\hat{\theta}} = 0 \end{aligned} \tag{43}$$

$$\frac{dh}{d\hat{\theta}} = -\frac{\xi(1-h-\delta)}{-m^2 \frac{\nu-\delta}{(1-h-\delta)^2} + 1 - h - \delta - 3\xi(\hat{\theta} - \underline{\theta})} > 0 \tag{44}$$

Participation costs m

$$\begin{aligned} &- 2m(\nu-\delta)\nu + 2m(\nu-\delta)(1-h) - m^2(\nu-\delta)\frac{dh}{dm} \\ &- 3\hat{\theta}\xi(1-h-\delta)^2\frac{dh}{dm} + (1-h-\delta)^3\frac{dh}{dm} + 3(1-h)(1-h-\delta)^2\frac{dh}{dm} = 0 \end{aligned} \tag{45}$$

$$\frac{dh}{dm} = \frac{2m(\nu-\delta)\frac{\nu-(1-h)}{(1-h-\delta)^2}}{-m^2\frac{\nu-\delta}{(1-h-\delta)^2}+1-h-\delta-3\xi(\hat{\theta}-\underline{\theta})} < 0 \tag{46}$$

Holdouts payments

$$\begin{aligned}\frac{dh}{d\nu} &= -m^2(\nu-\delta)-m^2\nu+m^2(1-h)-m^2(\nu-\delta)\frac{dh}{d\nu}-3\hat{\theta}\xi(1-h-\delta)^2\frac{dh}{d\nu} \\ &\quad +(1-h-\delta)^3\frac{dh}{d\nu}+3(1-h)(1-h-\delta)^2\frac{dh}{d\nu}=0\end{aligned} \tag{47}$$

$$\frac{dh}{d\nu} = \frac{m^2\frac{2\nu-\delta-(1-h)}{(1-h-\delta)^2}}{-m^2\frac{\nu-\delta}{(1-h-\delta)^2}+1-h-\delta-3\xi(\hat{\theta}-\underline{\theta})} < 0 \tag{48}$$

Implicit in the corresponding signs above is that denominator is non positive. So now, let us see what conditions on the parameters do we need for that.

First of all, if the second member of denominator is negative, we would obtain a non positive level due to:

$$\underbrace{-m^2}_{<0}\underbrace{(\nu-\delta)}_{>0}+(1-h-\delta-3\xi(\hat{\theta}-\underline{\theta}))\underbrace{(1-h-\delta)^2}_{>0}$$

Then we need:

$$\hat{\theta} > \frac{1-h-\delta}{3\xi}+\underline{\theta}$$

But in case $\hat{\theta}-\underline{\theta}$ are not big enough, we need a second condition for a non positive denominator:

$$\hat{\theta} > \frac{1-h-\delta-m^2\frac{\nu-\delta}{(1-h-\delta)^2}}{3\xi}+\underline{\theta}$$

So having the second, we ensure the first one.

I Social utility maximizing haircut

In this context, social welfare is the aggregate expected utility of both investors and government:

$$W^*(h) = \int_{\check{\theta}}^{\theta^*(h)} (\theta+0)f(\theta)d\theta + \int_{\theta^*(h)}^{\hat{\theta}} (\theta(1+\xi)-(1-h)+1-h-m)f(\theta)d\theta \quad (49)$$

$$W^*(h) = \int_{\check{\theta}}^{\hat{\theta}} (\theta)f(\theta)d\theta + \int_{\theta^*(h)}^{\hat{\theta}} (\theta\xi - m)f(\theta)d\theta \quad (50)$$

Manipulating the expression above, we can use expectation and truncated variable expectation formulas to ease optimum estimation.

$$W^*(h) = E[\theta] + \xi E[\theta|\theta > \theta^*(h)] - m(1 - F(\theta^*(h))) \quad (51)$$

$$W^*(h) = \frac{\check{\theta} + \hat{\theta}}{2} + \xi\frac{\theta^*(h) + \hat{\theta}}{2} - m\frac{\hat{\theta} - \theta^*(h)}{\hat{\theta} - \check{\theta}} \quad (52)$$

Now we take the first derivative of $W^*(h)$ respect to haircut to get the haircut that maximizes expected social utility in (55).

$$\frac{\partial W^*(h)}{\partial h} = \frac{\xi}{2}\frac{\partial\theta^*(h)}{\partial h} + \frac{m}{\hat{\theta} - \check{\theta}}\frac{\partial\theta^*(h)}{\partial h} = 0 \quad (53)$$

$$\frac{\partial W^*(h)}{\partial h} = \left(\frac{\xi}{2} + \frac{m}{\hat{\theta} - \check{\theta}}\right)\left(-1 + \frac{m(\nu - \delta)}{(1 - h - \delta)^2}\right) = 0 \quad (54)$$

$$h_{soc} = 1 - \delta - \sqrt{m(\nu - \delta)} \quad (55)$$

J Nash bargaining haircut

In this section we introduce our setting into a Nash generalized bargaining process, to now obtain the haircut as the allocation that maximizes Nash product in (56). $\omega(h)$ represents net output (restructuring success against failure) to distribute between agents: government and bondholders. Each of them has some bargaining power α and $1 - \alpha$ respectively which will determine its share of the total product.

$$\omega(h) = (\bar{\theta}(1+\xi) - [(1-h)\ell + \nu(1-\ell)] - \theta)^{\alpha}(1-h-m-\delta\nu)^{1-\alpha} \tag{56}$$

It is important to note that if one investor i does not participate, it would not change the result of the agreement. So its individual outside option is to receive $\nu\delta$ and not zero (as the case in which the proposal fails).

We will obtain the haircut as the allocation that maximizes generalized Nash product $\omega(h)$ expected in θ.

$$h = argmax\{\omega(h)\}$$

Now taking the derivative of (56) in h.

$$\alpha\ell(\xi\bar{\theta} - (1-h)\ell - \nu(1-\ell))^{\alpha-1}(1-h-m-\delta\nu)^{1-\alpha} = \\ (1-\alpha)(\xi\bar{\theta} - \nu - \ell(1-h-\nu))^{\alpha}(1-h-m-\delta\nu)^{-\alpha} \tag{57}$$

$$\alpha\ell(1-h-m-\delta\nu) = (1-\alpha)(\xi\bar{\theta} - \nu - \ell(1-h-\nu)) \tag{58}$$

$$\alpha\ell(1-m-\delta\nu) - (1-\alpha)(\xi\bar{\theta} - \nu - \ell(1-\nu)) = (1-\alpha)\ell h + \alpha\ell h \tag{59}$$

$$h = \frac{\alpha\ell(1-m-\delta\nu) - (1-\alpha)(\xi\bar{\theta} - \nu - \ell(1-\nu))}{\ell} \tag{60}$$

We can get the α value for the eqiuilibrium h as well.

$$\alpha = \frac{\xi\bar{\theta} - \nu - \ell(1-h-\nu)}{(\ell(-m-\delta\nu) + \xi\bar{\theta} - \nu - \ell(-\nu))} \tag{61}$$

With $\ell = 1$:

$$\alpha = \frac{\xi\bar{\theta} - (1-h)}{\xi\bar{\theta} - (m+\delta\nu)} \tag{62}$$